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# Comparative Analysis of Conventional and Non-Conventional Optimisation Techniques for CNC Turning Process

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*This paper describes various optimisation procedures for solving the CNC turning problem to find the optimum operating parameters such as cutting speed and feedrate. Total production time is considered as the objective function, subject to constraints such as cutting force, power, tool–chip interface temperature and surface roughness of the product. Conventional optimisation techniques such as the Nelder Mead simplex method and the boundary search procedure, and non-conventional techniques such as genetic algorithms and simulated annealing are employed in this work. An example is given to illustrate the working procedures for determining the optimum operating parameters. Results are compared and their performances are analysed.*

**Keywords:** Boundary search procedure; Genetic algorithm; Nelder Mead simplex method; Optimisation; Simulated annealing

## 1. Introduction

Optimisation of operating parameters is an important step in machining optimisation, particularly for operating CNC machine tools. With the general use of sophisticated and high-cost CNC machines coupled with higher labour costs, optimum operating parameters are desirable for producing the part economically. Although there are handbooks that provide recommended cutting conditions, they do not consider the economic aspect of machining and also are not suitable for CNC machining. The operating parameters in this context are cutting speed, feedrate, depth of cut, etc., that do not violate any of the constraints that may apply to the process and satisfy objective criteria such as minimum machining time or minimum machining cost or the combined objective function of machining time and cost.

Machining optimisation problems have been investigated by many researchers. Gilbert [18] presented a theoretical analysis of optimisation of the process by using two criteria: maximum production rate and minimum machining cost. Since the results obtained by using these two different criteria are always different, a maximum profit rate, which yields a compromise result has been used in subsequent investigations. These early studies were, however, limited to single-pass operations without consideration of any constraints. Subsequent studies included various operating constraints in the optimisation models and several techniques have been tried for optimisation [1–7]. In order to account for the stochastic nature of tool use, various probabilistic approaches have also been proposed. Since multipass operation is often preferred to single-pass operation for economic reasons, efforts have been made to determine optimal machining conditions for multipass operations [2,8,9].

So far, machining optimisation problems have been investigated by many researchers using conventional techniques such as geometric programming [3,4], convex programming [10], and the Nelder Mead simplex search method [1,2]. Ruy Mesquita et al. [7] have also used a conventional technique (direct search method – combination of Hook and Jeeves and random search) for optimisation of multipass turning. A mathematical model proposed by S. J. Chen was used in this work. It is observed that the conventional methods are not robust because:

1. The convergence to an optimal solution depends on the chosen initial solution.
2. Most algorithms tend to become stuck on a suboptimal solution.
3. An algorithm efficient in solving one machining optimisation problem may not be efficient in solving a different machining optimisation problem.
4. Computational difficulties in solving multivariable problems (more than four variables).
5. Algorithms are not efficient in handling multiobjective functions.
6. Algorithms cannot be used on a parallel machine.

To overcome the above problems, non-conventional techniques are used in this work for solving the optimisation of

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the turning process. In earlier work, a standard optimisation procedure using a genetic algorithm [11] was developed to solve different machining optimisation problems such as turning, face milling and surface grinding [12]. In this work, in addition to a GA, the simulated annealing technique has been tried.

In this work, the mathematical model proposed by Agapiou [1,2] is adopted for finding the optimum operating parameters for CNC turning operations. Total production time is considered as the objective function, subject to constraints such as cutting force, power, tool–chip interface temperature, and surface roughness of the product. Conventional optimisation techniques such as the Nelder Mead simplex method and the boundary search procedure, and non-conventional techniques such as genetic algorithms and simulated annealing are used in this work. An example is given to illustrate how these procedures are used to determine the optimum operating parameters. Results are compared and their performances are analysed.

## 2. The Optimisation Problem [1]

### 2.1 Objective Function

The total time required to machine a part is the sum of the times necessary for machining, tool changing, tool quick return, and workpiece handling:

$$T_u = t_m + t_{cs}\,(t_m/T) + t_R + t_h$$

The cutting time per pass ($t_m$) = $\Pi DL/1000\ V\ f$. Taylor's tool life equation is:

$$V\, f^{a1}\, \mathrm{doc}^{a2}\, T^{a3} = K$$

This Eq. is valid over a region of speed and feed for which the tool life ($T$) is obtained.

### 2.2 Operating Parameters

#### 2.2.1 Feedrate

The maximum allowable feed has a pronounced effect on both the optimum spindle speed and production rate. Feed changes have a more significant impact on tool life than depth of cut changes. The system energy requirement reduces with feed, since the optimum speed becomes lower. Therefore, the largest possible feed consistent with the allowable machine power and surface finish is desirable, in order for a machine to be fully used. It is often possible to obtain much higher metal removal rates without reducing tool life by increasing the feed and decreasing the speed. In general, the maximum feed in a roughing operation is limited by the force that the cutting tool, machine tool, workpiece and fixture are able to withstand. The maximum feed in a finish operation is limited by the surface finish requirement and can often be predicted to a certain degree, based on the surface finish and tool nose radius.

#### 2.2.2 Cutting Speed

Cutting speed has a greater effect on tool life than either depth of cut (or) feed. When compared with depth of cut and feed, the cutting speed has only a secondary effect on chip breaking, when it varies in the conventional speed range. There are certain combinations of speed, feed, and depth of cut which are preferred for easy chip removal which are mainly dependent on the type of tool and workpiece material. Charts providing the feasible region for chip breaking as a function of feed versus depth of cut are sometimes available from the tool manufacturers for a specific insert (or) tool, and can be incorporated in the optimisation systems.

### 2.3 Constraints

1. Maximum and minimum permissible feed rates, cutting speed, and depth of cut:

$$f_{\min} \le f \le f_{\max}$$
$$V_{\min} \le V \le V_{\max}$$
$$\mathrm{doc}_{\min} \le \mathrm{doc} \le \mathrm{doc}_{\max}$$

2. Power limitation:

$$0.0373\ V^{0.91}\ f^{0.78}\ \mathrm{doc}^{0.75} \le P_{\max}$$

3. Surface roughness limitations especially for finish pass:

$$14\ 785\ V^{-1.52}\ f^{1.004}\ \mathrm{doc}^{0.25} \le R_{a\max}$$

4. Temperature constraint:

$$74.96\ V^{0.4}\ f^{0.2}\ \mathrm{doc}^{0.105} \le \theta_{\max}$$

5. Cutting force constraint:

$$844\ V^{-0.1013}\ f^{0.725}\ \mathrm{doc}^{0.75} \le F_{\max}$$

## 3. Conventional Optimisation Techniques

### 3.1 Boundary Search Procedure (BSP)

To start with, the depth of cut (doc) is fixed and the feedrate ($f$) is assumed to be at the maximum allowable limit. The values of "doc" and "$f$" are substituted in power and temperature constraint equations and the corresponding cutting speeds ($V_p$ and $V_t$) are calculated. Further the obtained values "$f$" and "$V$" are accepted or modified according to constraint violation and objective function minimisation using the bounding phase and interval halving method the flowchart of which is given in Fig. 1.

### 3.2 Nelder Mead Simplex Method (NMS) [13]

In this method, three points are used in the initial simplex. At each iteration, the worst point ($X_h$) in the simplex is found. At first, the centroid ($X_o$) of all but the worst point is determined. Thereafter, the worst point in the simplex is reflected about the centroid and the new point ($X_r$) is found. If the constraints are not violated at this point the reflection is considered to have taken the simplex to a good region in the search space. Thus, an expansion ($X_e$) along the direction from the centroid to the reflected point is performed. On the other hand, if the constraints are violated at the reflected point, a contraction ($X_{\mathrm{cont}}$) in the direction from the centroid is made. The procedure

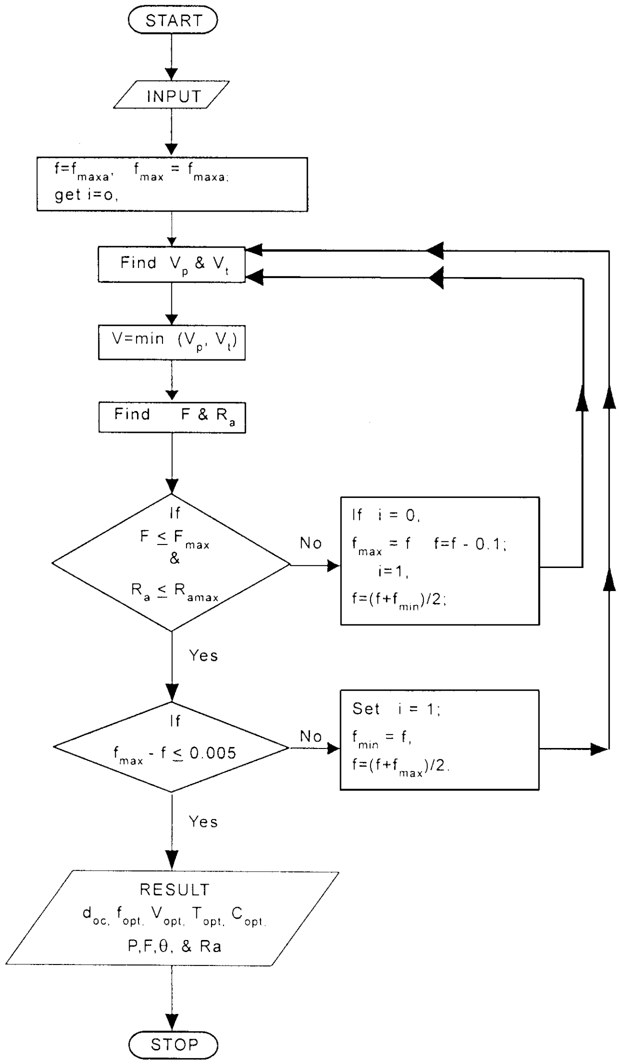


**Fig. 1.** Flowchart of "boundary search procedure".

**Table 1.** By boundary search procedure.

| Number | doc | $V$ | $f$ | $T_U$ |
|---|---|---|---|---|
| 1 | 2.0 | 119.77 | 0.762 | 2.84 |
| 2 | 2.5 | 112.95 | 0.762 | 2.93 |
| 3 | 3.0 | 114.02 | 0.680 | 3.11 |
| 4 | 3.5 | 118.28 | 0.582 | 3.34 |
| 5 | 4.0 | 122.13 | 0.509 | 3.59 |
| 6 | 4.5 | 124.45 | 0.452 | 3.84 |
| 7 | 5.0 | 125.10 | 0.406 | 4.10 |

is repeated until the termination criteria are met. The following equations are used in the method:

$$X_o = (X_l + X_g)/2$$
$$X_r = 2X_o - X_h$$
$$X_e = (1 + \gamma)\ X_o - \gamma\ X_h$$
$$X_{\text{cont}} = (1 + \beta)\ X_o - \beta\ X_h$$

**Table 2.** By Nelder Mead simplex method.

| Number | doc | $V$ | $f$ | $T_U$ |
|---|---|---|---|---|
| 1 | 2.0 | 118.32 | 0.750 | 2.87 |
| 2 | 2.5 | 113.13 | 0.738 | 2.97 |
| 3 | 3.0 | 108.67 | 0.660 | 3.15 |
| 4 | 3.5 | 117.50 | 0.546 | 3.44 |
| 5 | 4.0 | 126.25 | 0.473 | 3.69 |
| 6 | 4.5 | 125.42 | 0.463 | 3.84 |
| 7 | 5.0 | 126.35 | 0.420 | 4.10 |

Where, $\gamma$ = expansion coefficient (2.0); $\beta$ = contraction coefficient (0.5).

The flowchart is given in Fig. 2.

## 4. Non-Conventional Optimisation Techniques

### 4.1 Genetic Algorithms (GA) [14–17]

GAs form a class of adaptive heuristics based on principles derived from the dynamics of natural population genetics. The searching process simulates the natural evaluation of biological creatures and turns out to be an intelligent exploitation of a random search. A candidate solution (chromosomes) is represented by an appropriate sequence of numbers. In many applications the chromosome is simply a binary string of 0 and 1. The quality is the fitness function which evaluates a chromosome with respect to the objective function of the optimisation problem. A selected population of solution (chromosome) initially evolves by employing mechanisms modelled after those currently believed to apply in genetics. Generally, the GA mechanism consists of three fundamental operations: reproduction, crossover and mutation. Reproduction is the random selection of copies of solutions from the population according to their fitness value, to create one or more offspring. Crossover defines how the selected chromosomes (parents) are recombined to create new structures (offspring) for possible inclusion in the population. Mutation is a random modification of a randomly selected chromosome. Its function is to guarantee the possibility of exploring the space of solutions for any initial population and to permit the escape from a zone of local minimum. Generally, the decision of the possible inclusion of crossover/mutation offspring is governed by an appropriate filtering system. Both crossover and mutation occur at every cycle, according to an assigned probability. The aim of the three operations is to produce a sequence of populations that, on average, tends to improve.

#### *4.1.1 The Optimisation Procedure Using GA*

*Step 1*. Choose a coding to represent problem parameters, a selection operator, a crossover operator and a mutation operator. Choose a population size $n$, crossover probability $p_c$, and mutation probability $p_m$. Initialise a random population of strings of size l. Choose a maximum allowable generation number $t_{\max}$. Set $t = 0$.

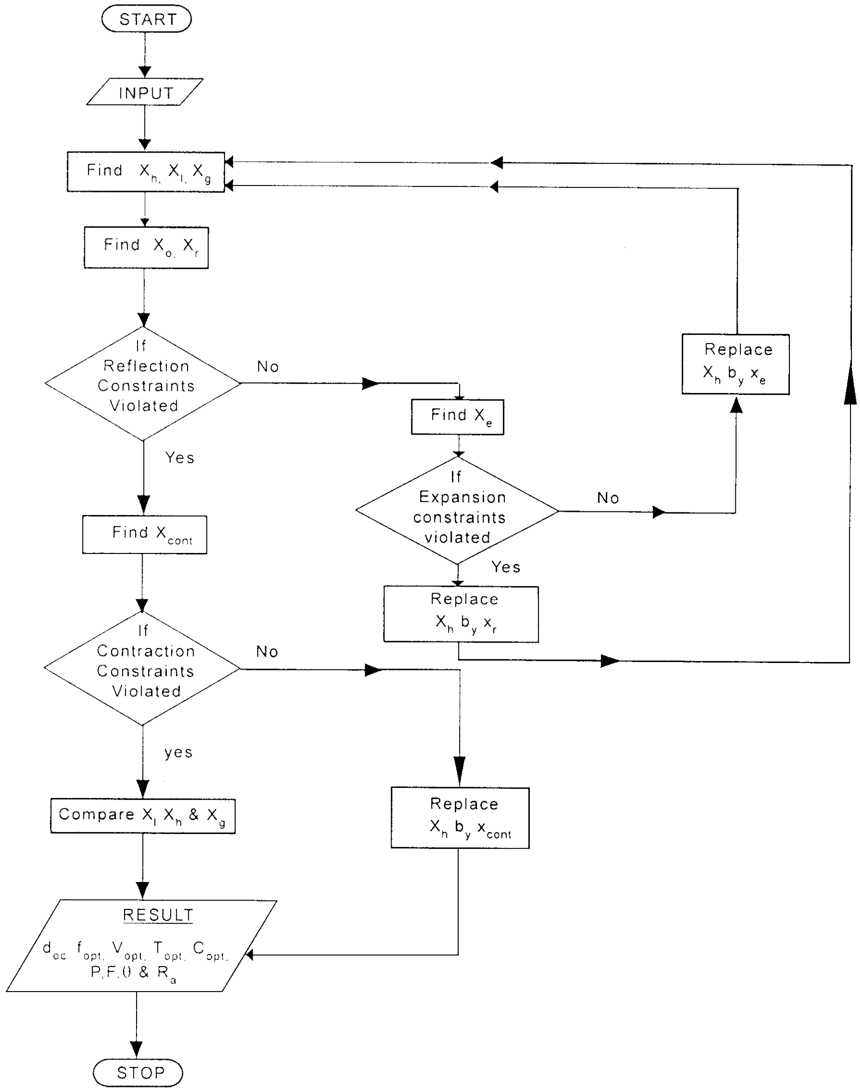


**Fig. 2.** Flowchart of "Nelder Mead simplex method".

*Step 2.* Evaluate each string in the population.

*Step 3.* If $t > t_{max}$ (or) other termination criteria is satisfied, terminate.

*Step 4.* Perform reproduction on the population.

*Step 5.* Perform crossover on random pairs of strings.

*Step 6.* Perform bitwise mutation.

*Step 7.* Evaluate strings in the new population. Set $t = t+1$ and go to step 3.

End

#### *4.1.2 GA Parameters*

| | |
|---|---|
| Sample size | = 20 |
| Crossover probability | = 0.8 |
| Mutation probability | = 0.05 |
| Number of generations | = 100 |

### 4.2 Simulated Annealing (SA) [16]

The simulated annealing procedure simulates the annealing process to achieve the minimum function value in a minimisation problem. The slow cooling phenomenon of the annealing process is simulated by controlling a temperature-like parameter introduced using the concept of the Boltzmann probability distribution. According to the Boltzmann probability distribution, a system at thermal equilibrium at a temperature $T$ has its energy distributed probabilistically according to $P(E)$ = exponent of $(-\Delta E/kT)$, where $k$ is the Boltzmann constant. This expression suggests that a system at a high temperature has an almost uniform probability of being at any energy state, but at a low temperature it has a small probability of being at a high energy state. Therefore, by controlling the temperature $T$ and assuming that the search process follows the Boltzmann probability distribution, the convergence of an algorithm can be controlled.

**Table 3.** By genetic algorithm.

| Number | doc | $V$ | $f$ | $T_U$ |
|---|---|---|---|---|
| 1 | 2.0 | 118.91 | 0.764 | 2.85 |
| 2 | 2.5 | 114.15 | 0.644 | 3.12 |
| 3 | 3.0 | 114.49 | 0.665 | 3.13 |
| 4 | 3.5 | 120.61 | 0.531 | 3.46 |
| 5 | 4.0 | 106.16 | 0.565 | 3.51 |
| 6 | 4.5 | 104.80 | 0.454 | 3.96 |
| 7 | 5.0 | 110.58 | 0.435 | 4.14 |

**Table 4.** By simulated annealing.

| Number | doc | $V$ | $f$ | $T_U$ |
|---|---|---|---|---|
| 1 | 2.0 | 120.39 | 0.753 | 2.85 |
| 2 | 2.5 | 112.57 | 0.761 | 2.93 |
| 3 | 3.0 | 116.68 | 0.648 | 3.15 |
| 4 | 3.5 | 118.21 | 0.582 | 3.34 |
| 5 | 4.0 | 122.05 | 0.507 | 3.59 |
| 6 | 4.5 | 125.41 | 0.448 | 3.85 |
| 7 | 5.0 | 126.28 | 0.400 | 4.12 |

The above procedure can be used in the function minimisation of the total production time in the CNC turning optimisation problem. The algorithm begins with an initial point $x^1$ ($V1$, $f1$) and a high temperature $T$. A second point $x^2$ ($V2$, $f2$) is created at random in the vicinity of the initial point and the difference in the function values ($\Delta E$) at these two points is calculated. If the second point has a smaller function value, the point is accepted; otherwise the point is accepted with a probability exp ($-\Delta E/T$). This completes one iteration of the simulated annealing procedure. In the next generation, another point is created at random in the neighbourhood of the current point and the Metropolis algorithm is used to accept or reject the point. In order to simulate the thermal equilibrium at every temperature, a number of points are usually tested at a particular temperature, before reducing the temperature. The algorithm is terminated when a sufficiently small temperature is obtained or a small enough change in function values is found.

### 4.2.1 The Optimisation Procedure Using SA

*Step 1*. Choose an initial point $x^1$ ($V1$, $f1$), a termination criteria $\epsilon$.

Set $T$ a sufficiently high value, number of iterations to be performed at a particular temperature $n$, and set $t = 0$.

*Step 2*. Calculate a neighbouring point $x^2$. Usually, a random point in the neighbourhood is created.

*Step 3*. If $\Delta E = E(x^2) - E(x^1) < 0$, set $t = t + 1$;
Else create a random number $r$ in the range (0,1). If $r \leq \exp(-\Delta E/T)$ set $t = t+1$;
Else go to step 2.

Step 4. If $|x^2 - x^1| < \epsilon$ and $T$ is small, terminate.
Else if ($t$ mod $n$) = 0 then lower $T$ according to a cooling schedule.
Goto step 2;

Else goto step 2.

## 5. Data of the Problem [1]

| | | |
|---|---|---|
| $L$ = 203 mm | $D$ = 152 mm | $V_{min}$ = 30 m min$^{-1}$ |
| $V_{max}$ = 200 m min$^{-1}$ | $f_{min}$ = 0.254 mm rev$^{-1}$ | $f_{max}$ = 0.762 mm rev$^{-1}$ |
| $R_{a\max}$ (rough) = 50 μm | $R_{a\max}$ (finish) = 20 μm | $P_{max}$ = 5 kW |
| $F_{max}$ = 900 N | $\theta_{max}$ = 500°C | $doc_{min(r)}$ = 2.0 mm |
| $doc_{max(r)}$ = 5.0 mm | $doc_{min(f)}$ = 0.6 mm | $doc_{max(f)}$ = 1.5 mm |
| $a1$ = 0.29 | $a2$ = 0.35 | $a3$ = 0.25 |
| $K$ = 193.3 | $t_{cs}$ = 0.5 min/edge | $t_R$ = 0.13 min/pass |
| $t_h$ = 1.5 min/piece | $C_o$ = $R_s$ 3.50/min | $C_t$ = $R_s$ 17.50/edge |

## 6. Optimisation Results

Tables 1 to 4 show the operating parameters such as cutting speed, feed rate and total production time obtained using conventional and non-conventional techniques.

Table 5 shows the total production time obtained from different techniques and percentage deviation of time with reference to BSP method.

## 7. Conclusion

In this work, optimisation of the CNC turning process is solved using conventional and non-conventional optimisation techniques. An example is given to illustrate the working procedures of these techniques in solving the above problem. It is observed that a boundary search procedure is able to find the exact answer, but it is not flexible enough to include more

**Table 5.** The total production time obtained from different techniques and percentage deviation of time with reference to BSP methods.

| | | | NMS | | GA | | SA | |
|---|---|---|---|---|---|---|---|---|
| Number | doc | BSP $T_U$ | $T_U$ | % dev | $T_U$ | % dev | $T_U$ | % dev |
| 1 | 2.0 | 2.84 | 2.87 | +1.0 | 2.85 | +0.4 | 2.85 | +0.4 |
| 2 | 2.5 | 2.93 | 2.97 | +1.0 | 3.12 | +1.0 | 2.93 | 0 |
| 3 | 3.0 | 3.11 | 3.15 | +1.0 | 3.13 | +1.0 | 3.15 | +1.0 |
| 4 | 3.5 | 3.34 | 3.44 | +3.0 | 3.46 | +3.0 | 3.34 | 0 |
| 5 | 4.0 | 3.59 | 3.69 | +3.0 | 3.51 | +3.0 | 3.59 | 0 |
| 6 | 4.5 | 3.84 | 3.88 | +1.0 | 3.96 | +1.0 | 3.85 | +0.3 |
| 7 | 5.0 | 4.10 | 4.23 | +3.0 | 4.14 | +3.0 | 4.12 | +0.5 |

variables and equations. The results obtained by simulated annealing are comparable with the boundary search procedure and the flexibility of the method needs further investigation. The Nelder Mead simplex method deviates from the boundary search method by 1%–3%, but with simple modification it can be extended to other machining problems by including one more variable. The genetic Algorithm method also deviates from boundary search method by 1%–3%, but by adopting a suitable coding system, this can be used to solve any type of machining optimisation problem such as milling, cylindrical grinding, surface grinding, etc., by including any number of variables.

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## Nomenclature

| | |
|---|---|
| $D$ | diameter of the workpiece (mm) |
| $L$ | length of the workpiece (mm) |
| $V$ | cutting speed (m min$^{-1}$) |
| $V_{\min}$, $V_{\max}$ | minimum and maximum allowable cutting speeds |
| $f$ | feedrate (mm rev$^{-1}$) |
| $f_{\min a}$, $f_{\max a}$ | minimum and maximum allowable feedrates |
| $R_a$ | surface roughness ($\mu$m) |
| $R_{a\max(r)}$, $R_{a\max(f)}$ | maximum surface roughness of rough and finish cut |
| $P$ | power of the machine (kW) |
| $F$ | cutting force (N) |
| $\theta$ | temperature of tool–workpiece interface (°C) |
| doc | depth of cut (mm) |
| $\text{doc}_{\min(r)}$, $\text{doc}_{\max(r)}$ | minimum and maximum allowable depth of cut (rough) |
| $\text{doc}_{\min(f)}$, $\text{doc}_{\max(f)}$ | minimum and maximum allowable depth of cut (finish) |
| $a1$, $a2$, $a3$, $K$ | constants used in tool life equation |
| $T$ | tool life (min) |
| $t_m$ | machining time (min) |
| $t_{cs}$ | tool change time (min/edge) |
| $t_R$ | quick return time (min/pass) |
| $t_h$ | loading and unloading time (min/pass) |
| $T_u$ | total production time (min) |
| $C_o$ | operating cost ($R_s$/piece) |
| $C_t$ | tool cost per cutting edge ($R_s$/edge) |
| $C_T$ | total production cost($R_s$/piece) |
| $X_l$ | best point in the simplex |
| $X_g$ | next to best point in the simplex |
| $X_h$ | worst point in the simplex |